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OPTIMAL SCHEDULING OF MINING OPERATIONS TO IMPROVE PRODUCT QUALITY

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Abstract

Industries in Asia are creating a strong demand for commodities like copper, nickel, aluminium and iron ore thus making mining industry a substantial contributor to the world economy. In response to the increasing demand, it is important to run the existing operations in maximum capacity. This paper focuses on maximizing the operational capacity and thus increasing the profit of a mining industry using mathematical optimization techniques. The demand of the mining products are in a specified quality window, deviation from which would cost the industry a predefined contractual penalty. Material of different quality specifications are available at the stockpile which are blended in order to meet with the demand thus making this problem a Generalized Pooling Problem (GPP). The mathematical model is essentially a Mixed Integer Non Linear Programming Model (MINLP). A Multi Operations Sequencing and a continuous time mathematical model is proposed to schedule the operations in the mining industry. In this problem we considered stockpiles, stockyards, and vessel hatch as three different stages, where the blending occurs in stockyard and vessel hatch before the product reaches the customer.

Keywords - Optimization, MINLP, Mining, blending, Linear Programming, Bilinear programming

1. INTRODUCTION

Minerals is the back-bone of economic growth of any nation, and the quality and quantity and timely extraction of minerals depends upon the supply chain of the mining industry. The paper aims at solving the Mining Operations Scheduling Problem from the raw materials to the customer end. The intermediate processes include the raw material and finished goods stock blending. The problem considered is an example based upon real world problem consisting of Stockpiles, Stockyards, Vessel Hatch and Customer, all of which are collectively called resources. Fig. 1 shows schematic of a mining supply chain from raw material to customer end. The operations are basically the flow of materials in this case raw materials and finished goods from one resource to another. The blending takes place in the resources and the blend is subsequently transported further. The formulation and methodology has been based upon Blend Oil Operations Scheduling solution put forth by Grossman [1]. Boland, Natashia, et al. have carried out research and obtained different solutions for this problem in their paper [2].

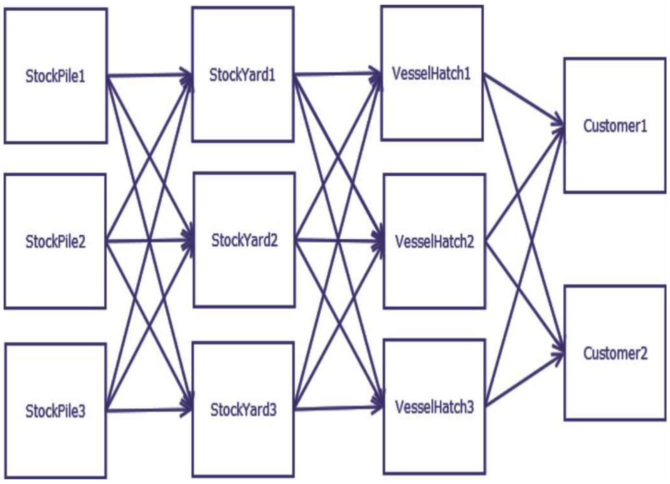

Fig. 1 Schematic of a mining supply chain

The features of Mining supply chain blending model can be summed in the points below:

- After the raw material is processed it is stored in stockpiles and later is transported into individual storages. All these storage are collectively termed as resources. These storage points are denoted by $R = \{1, \ldots, R\}$
- Different stockpiles are denoted by different properties and the stockpiles are considered as basic blends. Later in all the resources the blend is described as the mixture of basic blends. The properties of materials are denoted by $Q = \{1, \ldots, Q\}$.
- The operations flowing from the resources are combinatorial sets of all the connections between the resources, which are directed forward, these sets are denoted by $O = \{1, \ldots, O\}$
- Time slots in the formulation, denoted by $T = \{1, \ldots, T\}$
- The start time and duration of the operations to be scheduled are denoted by $S_{o,i}$ *and* $D_{o,i}$, these times are attributed to Operations and Time slots.

2. METHODOLOGY

An optimization model is a mathematical programming model consisting of a function, essentially the objective function which can either be maximized or minimized. This algorithm was originally put forth by Dantzig [3]. The constraints make up the feasible area for the objective function and the solution of this model. The following is a standard form of Optimization Model.

$$\min c^T x$$
Such that,
$$Ax \leq B, x_i \geq 0$$

With $x = (x_1, \ldots, x_n)$ $x = (x_1, \ldots, x_n)$ the variables of the problem, $c = (c_1, \ldots, c_n)$ $c = (c_1, \ldots, c_n)$ are the coefficients of the objective function, A is $p \times n$ matrix, and b constants with $b \geq 0$ MINLP also finds the integer solution of the problem. Solving for the integer solutions makes the problem essentially a NP-Hard problem. This solution uses branch and cut algorithm for solving integer linear programming, originally put by Padberg et. al. [4]

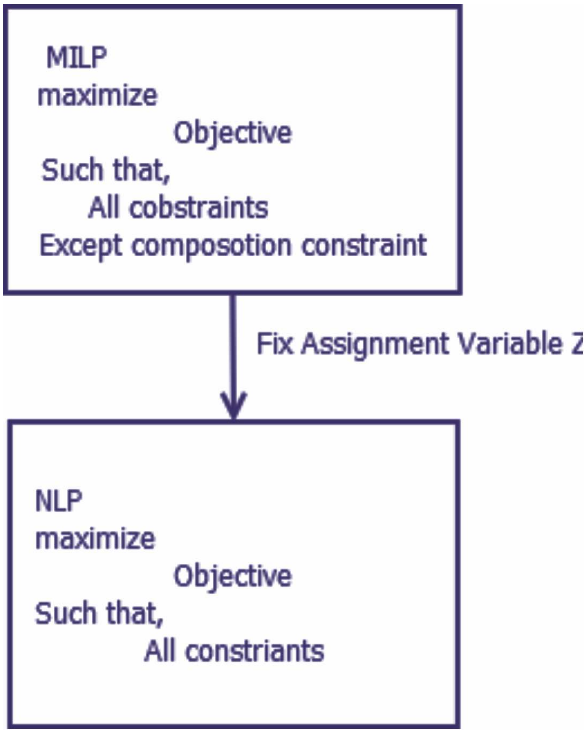

Fig. 2 Two-step decomposition method

The general method for solving the problem as mentioned before is based upon the Simplex method. But one of the shortcomings of the Simplex method is that it can solve only the linear problems, and as soon as there is some non-linearity in the constraints one needs a different method to solve the problem. This can be done by solving the problem using constraint programming solver. The technique introduced for solving MINLP is a two-step decomposition method, wherein first MILP problem is solved, excluding the nonlinear constraint seen in equation (10).The solution from MILP is then passed into the NLP model as a starting solution, where all the constraints are considered. NLP model is solved using CPLEX's constraint programming (CP)solver [5]. This is depicted in fig. 2.

3. FORMULATION

The formulations is designed keeping the real life scenario in mind, the notations used in the equations are described in Table 2. The formulation in the paper is based upon continuous time scheduling formulation which uses time slots for formulating the model. The continuous time representation model was introduced by Mock us et. al. [6].

Table 1. Notations and their explanations

Sets and Indices	
$i \in T := \{1, \ldots, T\}$	Set of Time slots
$q \in Q := \{1, \ldots, Q\}$	Set of Properties
$r \in R := \{1, \ldots, R\}$	Set of Resources
. $\in O = \{1,\ldots,O\}$	Operation
$b \in B = \{1,\ldots,B\}$	Set of Blends

Parameters	
$IVol_{r,b}$	Initial volume of minerals present in resource $r \in R$ of blend $b \in B$
$PropBlend_{b,q}$	Values of property $q \in Q$ in blend $b \in B$
O_{maxvol} and O_{minvol}	Maximum and minimum allowable values to flow through a particular operation $o \in O$
O_{maxfr} O_{minfr}	Maximum and minimum allowable flow rate of operation $o \in O$
$Propmin_{o,q}$ $Propmax_{o,q}$	Minimum and Maximum allowable constraints on property $q \in Q$ at operation $o \in O$

Variables	
$Z_{o,i}$	Binary assignment variable indicating whether a particular operation $o \in O$ is active at a particular time slot $i \in T$
$S_{o,i}$	Start time of operation $o \in O$ active in the time slot $i \in T$
$D_{o,i}$	Duration of operation $o \in O$ active in the time slot $i \in T$
$TVol_{o,i}$	Total volume of minerals flowing through operation $o \in O$ at time slot $i \in T$
$Vol_{o,b,i}$	Volume of a particular blend $b \in B$ flowing during the operation $o \in O$ at time slot $i \in T$
$TL_{r,i}$	Total level of mineral present in resource $r \in R$ at time $i \in T$
$TL_{r,i}$	Level of individual blend $b \in B$ of mineral present in resource $r \in R$ at time $i \in T$

3.1. Objective Function

The main objective function is to maximize the profit of the mining process. Here the Gross Margin (GM) of the minerals is considered. There can be many different objectives that can be considered in this problem like the inventory cost, the cost of demurrage of the vessel while unloading. Here the primary focus is on maximizing the profit using the GM of the mineral.

$$Max \sum_{o,b,i} GM_b \times V_{o,b,i} \quad \forall\, o \in o_{in} \mid o_{in} = r_c, b, i \tag{1}$$

Equation (1) denotes the maximizing expressions which states that the product of Gross Margin (GM) and the volume of each blend flowing through the operations, the input of which is connected to the customer resource, should be maximized.

3.2. Constraints

Equation (2) assigns start time and duration for every operation, and it states that the sum of start time and duration must not exceed the total time horizon. And the binary variable *Z* is multiplied with the time horizon to make sure the times are allotted only if the particular operation is active. It is essential that the formulations keeps track that the sum of all the individual blends in operations and resources should be equal to the total mixture of the blends in the same. This is ensured by the constraint stated in equation (3) and (4). In the real life scenarios, in all the operations blend mix is transferred using trucks or conveyors. Thus there is a maximum and minimum limit of transportation of material that has to be taken into consideration. Equation (5) considers the maximum and minimum allowances of blends to be transferred through an operation. And equation (6) puts a limit on the rate of transfer of the blends in the same way. The blends flowing into one resource should be equal to the blend that is already into the resource plus the crude that flows out of the resource, this is essentially a mass balance constraint as shown in equation (7) and (8). Property constraint ensures that the property at the demand side is within the specified constraints. Equation (9) denotes the property constraint. The proportion of a particular blend inside a resource should be equal to the proportion of the same blend flowing out of the resource. This equation is what brings non-linearity into the model and thus makes it even more difficult to solve. Equation (10) is known as the composition constraint.

$$S_{o,i} + D_{o,i} \le H \times Z_{o,i} \tag{2}$$

$$\sum_{b} V_{o,b,i} = TV_{o,i} \; \forall\, o, b, i \tag{3}$$

$$\sum_{b} L_{r,b,i} = TL_{r,i} \; \forall\, r, b, i \tag{4}$$

$$O_{minvol} \le TV_{o,i} \le O_{maxvol} \quad \forall\, o, i \tag{5}$$

$$O_{minfr} \le TV_{o,i} \le O_{maxfr} \quad \forall\, o, i \tag{6}$$

$$L_{r,b,i} = IVol_{r,b} - \sum_{o_{out}, i \le i} Vol_{o,b,i} + \sum_{o_{in}, i \le i} Vol_{o,b,i} \;\; \forall\, b, i, o_{in} \mid o_{in} = r_{in}, o_{out} \mid o_{out} = r_{out} \tag{7}$$

$$TL_{r,i} = \sum_{b} IVol_{r,b} - \sum_{o_{out}, i \le i} TVol_{o,i} + \sum_{o_{in}, i \le i} TVol_{o,i} \;\; \forall\, i, o_{in} \mid o_{in} = r_{in}, o_{out} \mid o_{out} = r_{out} \tag{8}$$

$$Propmin_{o,q} \times TVol_{o,i} \le \sum_{b} PropBlend_{b,q} \times Vol_{o,b,i} \le Propmax_{o,q} \times TVol_{o,i} \; \forall\, o, q \tag{9}$$

$$Tvol_{o,i} \times L_{r,b,i} = TL_{r,i} \times Vol_{o,b,i} \; \forall r, b, o, i \tag{10}$$

Table 2. Resource Table

res_id	res_name	min_level	max_level
1	p1	0	100
2	p2	0	100
3	p3	0	100
4	p4	0	100
5	p5	0	100
6	p6	0	100
7	p7	0	100
8	s1	0	480
9	s2	0	540
10	s3	0	600
11	v1	0	490
12	v2	0	500
13	v3	0	550
14	c1	0	600
15	c2	0	600

Table 4. Stockpile volume

res_id	b_id	vol
1	1	60
2	2	40
3	3	30
4	1	60
5	2	50
6	3	20
7	2	60

Table 5. Initial Volume

res_id	b_id	vol
8	1	25
8	2	0
8	3	0
9	1	0
9	2	90
9	3	40
10	1	0
10	2	0
10	3	0
11	1	0
11	2	70
11	3	0
12	1	0
12	2	0
12	3	0
13	1	0
13	2	0
13	3	50

Table 6. Blends

blend_id	blend_name	GM
1	a	4.9
2	b	7.8
3	c	6.4

Table 3. Operations Table

opp_id	res_out	res_in	opp_name	volmax	flow_min	flow_max
1	1	8	p1s1	600	10	15
2	1	9	p1s2	600	10	15
3	2	8	p2s1	600	10	15
4	2	9	p2s2	600	10	15
5	3	8	p3s1	600	10	15
6	4	8	p4s1	600	10	15
7	4	9	p4s2	600	10	15
8	5	10	p5s3	600	10	15
9	6	10	p6s3	600	10	15
10	7	10	p7s3	600	10	15
11	8	11	s1v1	600	2	8
12	8	12	s1v2	600	2	8
13	8	13	s1v3	600	2	8
14	9	11	s2v1	600	2	8
15	9	12	s2v2	600	2	8
16	9	13	s2v3	600	2	8
17	10	11	s3v1	600	2	8
18	10	12	s3v2	600	2	8
19	10	13	s3v3	600	2	8
20	11	14	v1c1	600	1	6
21	12	14	v2c1	600	1	6
22	13	14	v3c1	600	1	6
23	11	15	v1c2	600	1	6
24	12	15	v2c2	600	1	6
25	13	15	v3c2	600	1	6

Table 9. Property Min / **Table 10. Property Max**

	prop_1	prop_2	prop_3	prop_1	prop_2	prop_3
20	1.8	27	61.2	2.5	37.5	68
21	1.8	27	61.2	2.5	37.5	68
22	1.8	27	61.2	2.5	37.5	68
23	1.8	27	61.2	2.5	37.5	68
24	1.8	27	61.2	2.5	37.5	68
25	1.8	27	61.2	2.5	37.5	68

Table 7. Properties

p_id	name
1	x
2	y
3	z

Table 8. Blend Property Value

	prop_1	prop_2	prop_3
blend_1	1.8	28.8	61
blend_2	2.5	37.5	69
blend_3	2.1	30.1	72.9

Table 5 states the initial volume in all the resources except the stockpile, which is considered as the supply. Table 6 shows the blends along with its names and the gross margin of each blend. Table 7 shows the properties of the blend. Table 8 shows the blend property value. Table 9 and table 10 shows the minimum and maximum allowable property of the blends for the operations directed towards customer.

3. RESULTS

The solution statistics of the defined problem are mentioned in the Table 11. The problem was solved using CPLEX MIP and CP solver. Thus this is the first successful research focused on using MOS technique, continuous time representation model, and two-step decomposition method for mining optimization problem. The problem consisted of 7 stockpile, 3 stockyard, 3 vessel hatch and 2 customer resources. In all there were 25 operations, and operations 20-25 were the ones directed towards the customer resources. The property are only active for the operations directed towards the customer.

Table 11. Solution Statistics

Constraints	Variables	MIP Objective	CP Objective	Gap %
2031	693	3941.2	2862.9	27.359

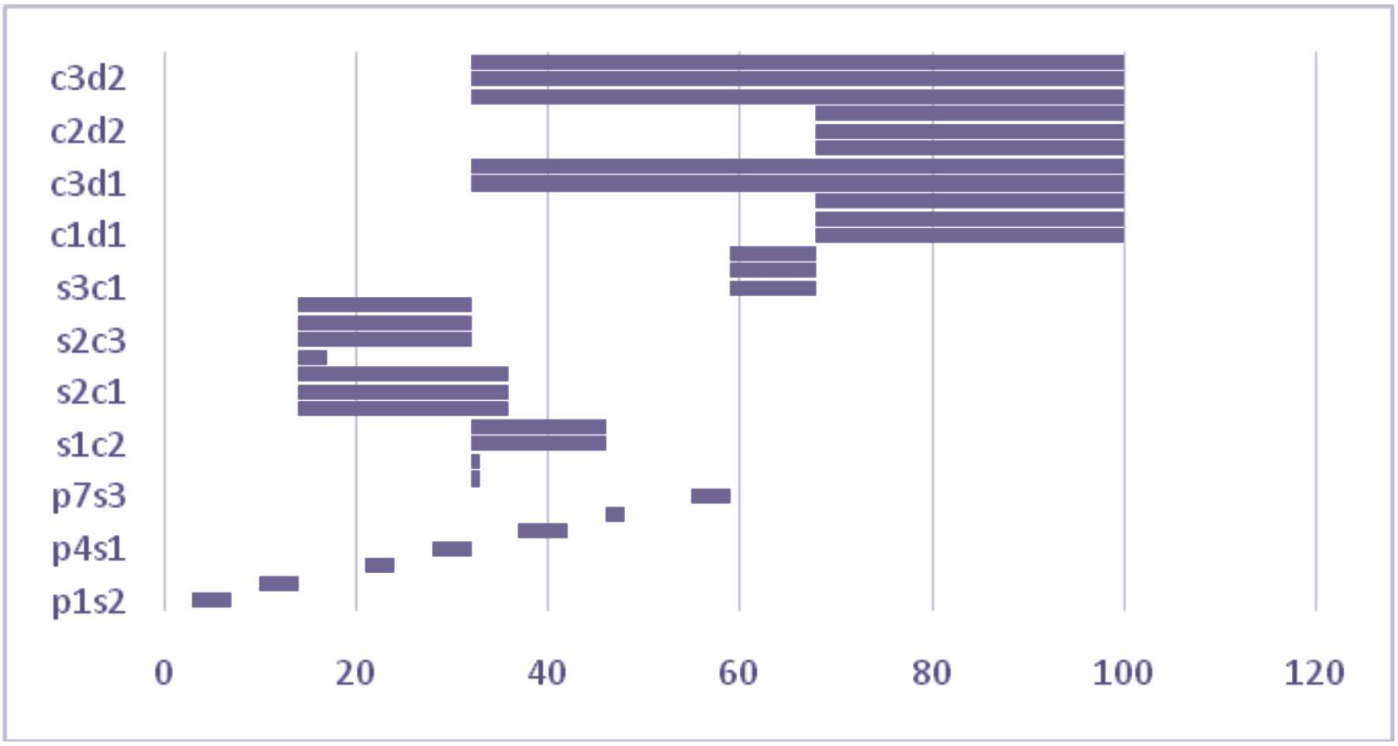

Fig. 3 Gantt chart showing schedule of the mining operations

5. CONCLUSION

In this paper a unified mining operations scheduling model using Multi operations sequencing time representations was presented, thus acquiring the most compact mathematical formulations as it requires fewer priority slots [1].With this formulation a MIP-CP gap of 27.359% for a problem of 2031 constraints and 693 variables. Both the MIP and NLP problems were solved using CPLEX version 12.7.1.0, the NLP problem was solved using the Constraint Programming feature of CPLEX.

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